General Mathematics UDF Guide

Version 1.4

LazyMath

Acknowledgements

A huge thank you to everyone who helped test the UDFs 😊

Contents

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Data Analysis

Summary Statistics

Determines the quartiles, fences, mean, and standard deviation of the input data.

Syntax

sum_stats(Data)

Where, *Data* represents a list containing the data to be analysed.

Example

The number of points a pro gamer scores on Flappy Bird over 10 games is shown in the table below.

Game	1	2	3	4	5	6	7	8	9	10
Score	12	47	58	73	20	31	10	22	17	250

Determine the quartiles, fences, and outliers (if any).

```
score_data                      {12.,47.,58.,73.,20.,31.,10.,22.,17.,250.}
summary_stats(score_data)
                                Total = 10
                                ▶Data Summary:
                                 "Minimum"        10.
                                    "Q1"          17.
                                    "Q2"          26.5
                                    "Q3"          58.
                                 "Maximum"        250.
                                   "IQR"          41.
                                "Lower Fence"     -44.5
                                "Upper Fence"     119.5
                                  "Range"         240.
                                  "Mean"          54.
                                "Standard Dev"    71.972
                                  "Skew"        "Positive"
                                Warning: Skew may be inaccurate
                                ▶Possible outliers:
                                {250.}
                                Sorted data saved as data.summary_stats
                                                                  Done
```

Warning: Skew may be inaccurate

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Dot Plot

Determines the summary statistics of an input dot plot.

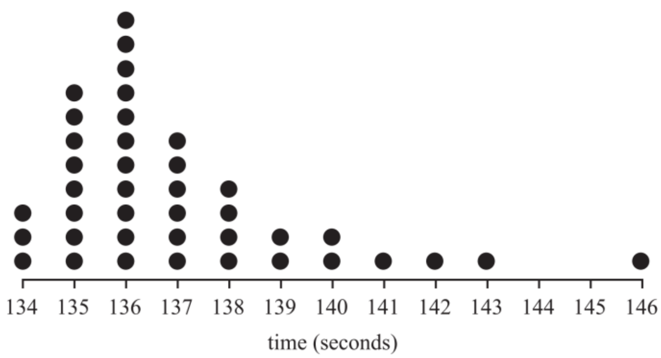
Syntax

dot_plot(Data)

Where, *Data* represents a matrix with the x-values in the top row, and the y-values in the bottom row.

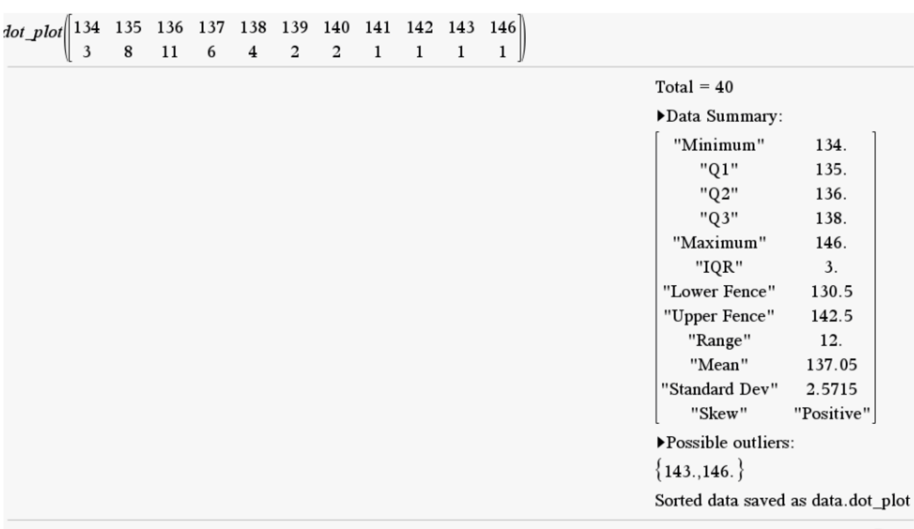
Example

The dot plot shows the times, in seconds, of 40 runners in the qualifying heats of their 800 m club championship.



Source: VCAA 2023 General Mathematics Written Examination 1 Question 1

Determine the median and skew of the data.



Warning: Skew may be inaccurate.

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Histogram

Determines the summary statistics of an input histogram.

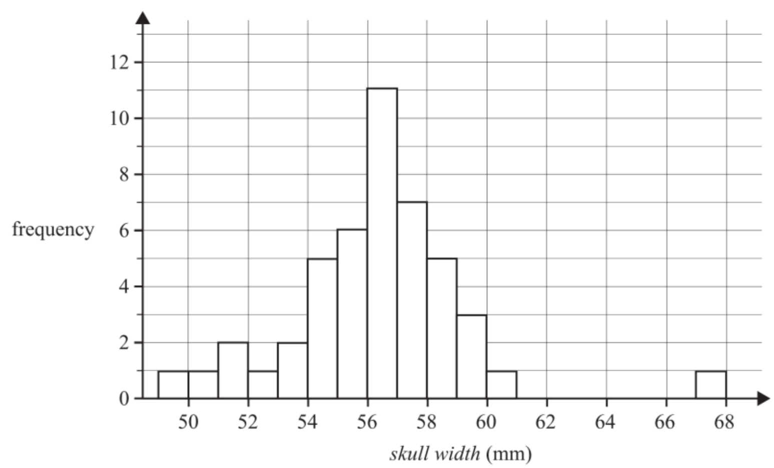
Syntax

histogram(Data)

Where, *Data* represents a matrix with the x-values in the top row, and the y-values in the bottom row.

Example

The histogram below displays the distribution of *skull width*, in millimeters, for 46 female possums.



Source: VCAA 2022 Further Mathematics Written Examination 1 Question 1

$$histogram\left(\begin{bmatrix} 49 & 50 & 51 & 52 & 53 & 54 & 55 & 56 & 57 & 58 & 59 & 60 & 67 \\ 1 & 1 & 2 & 1 & 2 & 5 & 6 & 11 & 7 & 5 & 3 & 1 & 1 \end{bmatrix}, 1\right)$$

Total = 46

▸Data Summary:

"Minimum"	"49–50"
"Q1"	"54–55"
"Q2"	"56–57"
"Q3"	"57–58"
"Maximum"	"67–68"
"IQR"	"2–4"
"Lower Fence"	"48–52"
"Upper Fence"	"60–64"
"Range"	"17–19"

▸Approximate values:

"Mean"	56.326
"Standard Dev"	2.9235
"Skew"	"Negative"

Warning: Skew may be inaccurate

▸Possible outliers:

{"49–50","67–68"}

Sample data saved as data.histogram

Done

Warning: Skew may be inaccurate.

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Frequency Table

Determines the frequency table of the input data list.

Syntax

freq_table(Data, *Minimum*, *Bin Size*)

Where,

Data represents a list containing the data

Minimum represents the starting point of the frequency table

Bin Size represents the size of each bin in the frequency table

Example

Determine the frequency table of the following data.

{35, 48, 45, 43, 38.2, 50, 39.8, 40.7, 40, 50, 35.4, 38.8, 40.2, 45, 45, 40, 43.3, 53.1, 35.6, 44.1, 34.8}

Start your table from 30 and use a bin size of 5.

freq_table({35,48,45,43,38.2,50,39.8,40.7,40,50,35.4,38.8,40.2,45,45,40,43.3,53.1,35.6,44.1,34.8},30,5)

▸Frequency Table:

"Interval"	"Frequency"	"Percentage"
"30−<35"	1.	4.7619
"35−<40"	6.	28.571
"40−<45"	7.	33.333
"45−<50"	4.	19.048
"50−<55"	3.	14.286
"Total"	21.	100.

Done

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Inverse Normal

Uses the 68-95-99.7% rule alongside the given mean and standard deviation to determine the values for which $Pr(X > x) = \%p$ and $Pr(X < x) = \%p$.

Syntax

norm_inverse(Mean, Standard Deviation, Percentage Probability)

Where,

Mean represents the mean of the normal distribution

Standard Deviation represents the standard deviation of the normal distribution

Percentage Probability represents the percentage probability of being less than or greater than a value

Example

The weight of dogs is normally distributed with a mean of 30 kg with a standard deviation of 3.4 kg.

Using the 68-95-99.7% rule, determine the weight which 16% of dogs are less than.

norm_inverse(30,3.4,16)

▸Given:

$\bar{x} = 30$ and sx = 3.4

▸Answer:

16% of values are less than 26.6

16% of values are greater than 33.4

Done

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Normal Bound

Uses the 68-95-99.7% rule to determine the cumulative percentage probability between two bounds, that is, $Pr(x_1 < X < x_2)$ %.

Syntax

norm_bound(Mean, Standard Deviation, Lower Bound, Upper Bound)

Where,

Mean represents the mean of the normal distribution

Standard Deviation represents the standard deviation of the normal distribution

Lower Bound represents the lower bound in the probability expression

Upper Bound represents the upper bound in the probability expression

Example

The weight of dogs is normally distributed with a mean of 30 kg with a standard deviation of 3.4 kg.

Using the 68-95-99.7% rule, determine the percentage of dogs which weigh between 26.6 kg and 36.8 kg.

norm_bound(30,3.4,26.6,36.8)

▸Given:

$\bar{x} = 30$ and $sx = 3.4$

▸Answer:

81.5% of the values are between 26.6 and 36.8

Done

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Normal Solve

Uses the 68-95-99.7% rule to determine the mean and standard deviation of a normal distribution, given two probabilities, $Pr(X < x_1) = p_1\%$ and $Pr(X > x_2) = p_2\%$.

Syntax

normsolve(Lower, % Pr(Lower), Upper, %Pr(Upper))

Where,

Lower represents the value, x_1

% Pr(Lower) represents the percentage probability of $X < x_1$, in other words, $p_1\%$

Upper represents the value, x_2

% Pr(Upper) represents the percentage probability of $X > x_2$, in other words, $p_2\%$

Example

The mean and standard deviation for the average weight of dogs is unknown.

After conducting some measurements, scientists determined that:

- 2.5% of dogs weigh more than 36.8 kg
- 16% of dogs weigh less than 26.6 kg

Use the 68-95-99.7% rule to determine, in kilograms, the mean and standard deviation.

```
norm_solve(26.6,16,36.8,2.5)
    ▶Given:
    16% of values are less than 26.6
    2.5% of values are greater than 36.8
    ▶Determine the number of sx from x̄ using 68−95−99.7% rule:
    26.6 is ⁻1 sx from x̄
    36.8 is 2 sx from x̄
    ▶Determine the equations:
    26.6 = x̄ − sx
    36.8 = x̄ + 2sx
    ▶Solve equations simultaneously for x̄ and sx:
    sx=3.4 and x̄=30.
                                                    Done
```

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Line Solve

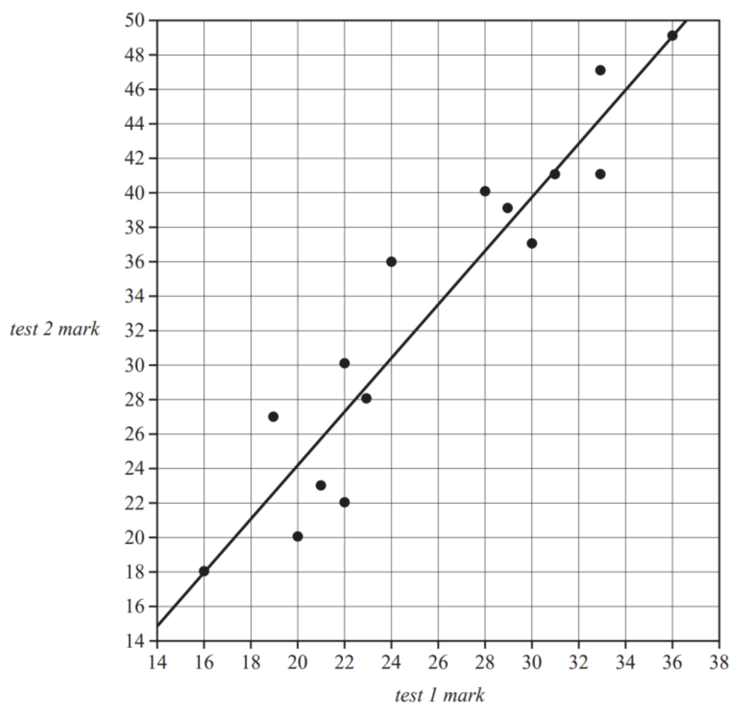
Determines the equation of the line passing through two input points.

Syntax

lin_solve(x1, y1, x2, y2)

Where, *x1*, *y1*, *x2*, *y2* represent the *x* and *y* coordinates of the two points respectively

Example



Source: VCAA 2023 General Mathematics Examination 2 Question 7 & Question 8

Determine the equation for the least squares line.

$lin\_solve(16,18,34,46) \quad y=1.5556\cdot x-6.8889$

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Linear Regression

Determines the least squares line, R, R^2, and association between the explanatory variable and the response variable.

Syntax

lin_reg(EV, RV)

Where,

EV represents a list containing the values of the explanatory variable

RV represents a list containing the values of the response variable

Example

The amount of money a student earns from their stocks each year is shown in the table below.

Year	1	2	3	4	5	6	7	8
Amount ($)	2.50	6.70	8.90	10.50	11.70	16.20	17.50	19.20

Determine the equation for the line of best fit of the data.

ev $\{1.,2.,3.,4.,5.,6.,7.,8.\}$

rv $\{2.5,6.7,8.9,10.5,11.7,16.2,17.5,19.2\}$

lin_reg(ev,rv)

Length = 8

▶Linear Regression:

$$\begin{bmatrix} \text{"Equation"} & y=2.3095\cdot x+1.2571 \\ \text{"R"} & 0.98899 \\ \text{"R}^2\text{"} & 0.9781 \\ \text{"Association"} & \text{"strong positive"} \\ \text{"Interpolation"} & \text{"}1\leq x\leq 8\text{"} \end{bmatrix}$$

Done

Linear Transformations

Determines the least squares line and R^2 of various transformations of the explanatory and response variables. These include squaring, reciprocal, and log10.

Syntax

lin_trans(EV, RV)

Where,

EV represents a list containing the values of the explanatory variable

RV represents a list containing the values of the response variable

Example

The amount of money a student earns from their stocks each year is shown in the table below.

Year	1	2	3	4	5	6	7	8
Amount ($)	2.50	6.70	8.90	10.50	11.70	16.20	17.50	19.20

Determine the least squares line with $log_{10}(amount)$ as the explanatory variable.

ev {1.,2.,3.,4.,5.,6.,7.,8.}

rv {2.5,6.7,8.9,10.5,11.7,16.2,17.5,19.2}

lin_trans(ev,rv)

▶Transforms:

"Trans"	"R²"	"Equation"
"x"	"0.9781"	"y=2.3095x+1.2571"
"x²"	"0.9104"	"y=0.24168x²+5.4871"
"y²"	"0.9556"	"y²=52.617x−72.426"
"log(x)"	"0.9375"	"y=18.13log(x)+1.2126"
"log(y)"	"0.8464"	"log(y)=0.10845x+0.51181"
"x⁻¹"	"0.7640"	"y=⁻17.02x⁻¹+17.432"
"y⁻¹"	"0.6002"	"y⁻¹=⁻0.0364x+0.29046"

Done

Residuals

Determines the least squares line fit and the differences between the true values and predicted values.

Syntax

residual(EV, RV)

Where,

EV represents a list containing the values of the explanatory variable

RV represents a list containing the values of the response variable

Example

The amount of money a student earns from their stocks each year is shown in the table below.

Year	1	2	3	4	5	6	7	8
Amount ($)	2.50	6.70	8.90	10.50	11.70	16.20	17.50	19.20

Determine the residual value for each year.

ev $\{1.,2.,3.,4.,5.,6.,7.,8.\}$

rv $\{2.5,6.7,8.9,10.5,11.7,16.2,17.5,19.2\}$

residual1(ev,rv)

"y"	"ŷ"	"Residual"
2.5	3.5667	⁻1.0667
6.7	5.8762	0.82381
8.9	8.1857	0.71429
10.5	10.495	0.00476
11.7	12.805	⁻1.1048
16.2	15.114	1.0857
17.5	17.424	0.07619
19.2	19.733	⁻0.53333

Done

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Mean Smoothing

Performs mean smoothing on the provided dataset and returns the result. Points which are marked with a blank string indicate they are invalid.

Syntax

mean_smooth(Data, Size)

Where,

Data represents a list containing the data to be mean smoothed

Size represents the group size which is used in smoothing

Example

The number of sales made by a company for the first eight months of 2025 is shown in the table below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Sales	200	250	100	350	450	500	890	320

Determine the four-mean smoothed data, with centering.

data $\{200.,250.,100.,350.,450.,500.,890.,320.\}$

mean_smooth$(data,4)$

$$\begin{bmatrix} \text{"x"} & \text{"y"} \\ 1. & \text{"}\square\text{"} \\ 2. & \text{"}\square\text{"} \\ 3. & 256.25 \\ 4. & 318.75 \\ 5. & 448.75 \\ 6. & 543.75 \\ 7. & \text{"}\square\text{"} \\ 8. & \text{"}\square\text{"} \end{bmatrix}$$

Done

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Median Smoothing

Performs median smoothing on the provided dataset and returns the result. Points which are marked with a blank string indicate they are invalid.

Syntax

med_smooth(Data, Size)

Where,

Data represents a list containing the data to be median smoothed

Size represents the group size which is used in smoothing

Example

The number of sales made by a company for the first eight months of 2025 is shown in the table below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Sales	200	250	100	350	450	500	890	320

Determine the three-median smoothed data.

data $\{200.,250.,100.,350.,450.,500.,890.,320.\}$

med_smooth(*data*,3)

$$\begin{bmatrix} \text{"x"} & \text{"y"} \\ 1. & \text{""} \\ 2. & 200. \\ 3. & 250. \\ 4. & 350. \\ 5. & 450. \\ 6. & 500. \\ 7. & 500. \\ 8. & \text{""} \end{bmatrix}$$

Done

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Seasonal Data

Determines the seasonal averages, seasonal indices, deseasonalised data, and the least square line fit of the deasonalised data. Rounding for each calculation step can be specified using the appropriate syntax.

Syntax

Case 1: Exact values

season(Data)

Where, *Data* represents the matrix containing the data, with each row representing one cycle and each column representing one period.

Note

In SACs and exams, you will have to round your answers at each stage. This case would be useful for checking your answers rather than obtaining the answers.

Case 2: Rounded values

season({"Data", Round_1, Round_2, Round_3, Round_4})

Where,

"Data" represents a string containing the name of the variable used to store the data

Round_1 represents the number of decimal places to round the average of each cycle to

Round_2 represents the number of decimal places to round the seasonal indices to

Round_3 represents the number of decimal places to round the average of the seasonal indices to

Round_4 represents the number of decimal places to round the deseasonilised data to

Note

All of the above must be inputted in sequence as a list

Example

The sales data for a clothing store was tracked quarterly for three years.

Year	2025				2026				2027			
Quarter	1	2	3	4	1	2	3	4	1	2	3	4
Sales	82	57	42	43	88	59	48	50	97	65	52	55

a. Calculate the sales average for each quarter. Give your answer correct to two decimal places.

b. Calculate the seasonal indices for each sale. Give your answer correct to three decimal places.

c. Calculate the average of the seasonal indices for each sale. Give your answer correct to two decimal places.

d. Deseasonalise the data. Give your answer correct to the nearest whole number.

e. Determine the least squares line fit for the deseasonalised data.

Case 1

data

82.	57.	42.	43.
88.	59.	48.	50.
97.	65.	52.	55.

season(data)

▸Season UDF:

▸Find averages for each cycle:

"Season"	1.	2.	3.	4.	"Avg"
"Cycle 1."	82.	57.	42.	43.	56.
"Cycle 2."	88.	59.	48.	50.	61.25
"Cycle 3."	97.	65.	52.	55.	67.25

▸Find indicies and take their average:

"Season"	1.	2.	3.	4.
"Cycle 1."	1.4643	1.0179	0.75	0.76786
"Cycle 2."	1.4367	0.96327	0.78367	0.81633
"Cycle 3."	1.4424	0.96654	0.77323	0.81784
"Avg"	1.4478	0.98256	0.76897	0.80068

▸Deseasonalise the data:

"Season"	1.	2.	3.	4.
"Cycle 1."	56.638	58.012	54.619	53.705
"Cycle 2."	60.782	60.048	62.421	62.447
"Cycle 3."	66.998	66.154	67.623	68.692

▸Find LSR fit of deseasonalised data:

y=1.3066x+53.019

Note: This may **not** provide the answers the marker will be looking for since it uses the exact value at each stage rather than the rounded values.

Case 2

data

$$\begin{bmatrix} 82. & 57. & 42. & 43. \\ 88. & 59. & 48. & 50. \\ 97. & 65. & 52. & 55. \end{bmatrix}$$

season({"data",2,3,2,0})

▶Season UDF:

▶Find averages for each cycle:

"Season"	1.	2.	3.	4.	"Avg"
"Cycle 1."	82.	57.	42.	43.	56.
"Cycle 2."	88.	59.	48.	50.	61.25
"Cycle 3."	97.	65.	52.	55.	67.25

▶Find indicies and take their average:

"Season"	1.	2.	3.	4.
"Cycle 1."	1.464	1.018	0.75	0.768
"Cycle 2."	1.437	0.963	0.784	0.816
"Cycle 3."	1.442	0.967	0.773	0.818
"Avg"	1.45	0.98	0.77	0.8

▶Deseasonalise the data:

"Season"	1.	2.	3.	4.
"Cycle 1."	57.	58.	55.	54.
"Cycle 2."	61.	60.	62.	63.
"Cycle 3."	67.	66.	68.	69.

▶Find LSR fit of deseasonalised data:

y=1.3007x+53.212

Significant Figures

Rounds an input number to a specific number of significant figures.

Syntax

sig_fig(Number, SF)

Where,

Number represents the number to round

SF represents the number of significant figures to round the number to

Example

Round the number 14.520010 to five significant figures.

sig_fig(14.52001,5) "14.520"

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Recursion and Financial Modelling

Recurrence Relation

Determines the compound interest per annum, annuity payment, and perpetuity payment of the input recurrence relation in the form

$$V_{n+1} = aV_n + b$$

Syntax

recur_rel(R, Pmt, V_0*, CpY)*

R represents the coefficient in front of V_n, that is *a* in the equation above

Pmt represents the amount being added to V_n, that is *b* in the equation above

V_0 represents the starting balance of the loan

CpY represents the number of periods per annum

Example

Let E_o = \$300 000 and $E_{n+1} = 1.003E_n - 2159.41$

a. Determine the compound interest per annum.

b. Determine the monthly payment, in dollars, the investor would receive if they wanted the annuity to act as a perpetuity.

Source: VCAA 2024 General Mathematics Written Examination 2 Question 7

```
recur_rel(1.003,-2159.41,3·10^5,12)
                              Reducing Balance Loan
                              ▸Recurrence Relation:
                              V(n+1) = 1.003Vn−2159.41, V₀ = 300000
                              ▸Interest:
                              I = (1.003 − 1) × 12 × 100 = 3.6
                              3.6% per annum, compounding monthly
                              ▸Payment: $2159.41 per month
                              ▸Interest Only: $900.00 per month
                                                                  Done
```

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Recurrence Relation Step

Displays the lines of working required to work out V_n given a recurrence relation in the form.

$$V_{n+1} = aV_n + b$$

Syntax

recur_rel_step(R, Pmt, V_0, Iter)

R represents the coefficient in front of V_n, that is *a* in the equation above

Pmt represents the amount being added to V_n, that is *b* in the equation above

V_0 represents the starting balance of the loan

Iter represents which term in the sequence we wish to determine

Example

Let E_o = \$300 000 and $E_{n+1} = 1.003E_n - 2159.41$

Showing recursive calculations, determine the balance of the annuity after two months. Round your answer to the nearest cent.

Source: VCAA 2024 General Mathematics Written Examination 2 Question 7

recur_rel_step$(1.003, {}^{-}2159.41, 3\cdot 10^5, 2)$

▶Recurrence Relation Step:
▶Working:
$V_0 = 300000$
$V_1 = 1.003\times300000-2159.41 = 298740.59$
$V_2 = 1.003\times298740.59-2159.41 = 297477.4$
▶Solution:
= \$297477.40

Done

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Amortisation Table

Generates the amortisation table based on the input payment amount, frequency of payments, interest rate, and starting balance.

Syntax

amor_tbl(%I, Pmt, V_0*, CpY, Iter)*

Where,

%I represents the percentage compound interest per annum

Pmt represents the payment per period

V_0 represents the starting balance of the loan

CpY represents the number of periods per annum

Iter represents the number of rows of the table you wish to generate

Example

Arthur invests $600 000 in an annuity that provides him with a monthly payment of $3973.00.
Interest is calculated monthly at a rate of 0.42% per month.
Complete the first four lines of the amortisation table. Round all values to the nearest cent.

Source: VCAA 2023 General Mathematics Written Examination 2 Question 6

amor_tbl$(0.42\cdot 12, -3973, 6\cdot 10^5, 12, 3)$

▸Recurrence Relation:

R = 1 + 5.04 / (12 × 100) = 1.0042

V(n+1) = 1.0042Vn−3973, V_0 = 600000

▸Amortisation Table:

"No."	"Pmt"	"I"	"PR"	"Bal"
0	"0.00"	"0.00"	"0.00"	"600000.00"
1	"3973.00"	"2520.00"	"1453.00"	"598547.00"
2	"3973.00"	"2513.90"	"1459.10"	"597087.90"
3	"3973.00"	"2507.77"	"1465.23"	"595622.67"

Reducing Balance Loan

Done

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Number of Payments

Determines the number of payments which could be made to pay off the loan and the final payment amounts.

Syntax

final_pmt(%I, Pmt, V_0*, CpY)*

Where,

%I represents the percentage interest per period

Pmt represents the payment per period

V_0 represents the starting balance of the loan
CpY represents the number of periods per annum

Example

Arthur borrowed $30 000 to buy a new motorcycle.

Interest on this loan is charged at a rate of 6.4% per annum, compounding quarterly.

Arthur will repay the loan in full using quarterly repayments of $1515.18. The final payment will differ slightly from the previous repayments.

Determine the total cost of repaying the loan, the final payment, and the number of payments required to pay off the loan.

Source: VCAA 2023 General Mathematics Written Examination 2 Question 5

final_pmt$(6.4,-1515.18,3\cdot 10^4,4)$

Reducing Balance Loan

I = 6.40%, Regular Pmt = $ 1515.18

PV = $ 30000.00

"No."	"Final Pmt"	"Total"
23	"3006.36"	"36340.32"
24	"1515.04"	"36364.18"

Done

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Finance Solver

Solves for a particular parameter based on the input values provided

Syntax

finance_solve(N, %I, PV, Pmt, FV, CpY)

N represents the number of payment periods

%I represents the interest rate per annum

PV represents the present value

Pmt represents the payment per period

FV represents the final value

CpY represents the number of periods per annum

Example

Bob has a student loan is $50,000 with an interest rate of 5.00% per annum, compounding monthly. Bob makes a payment of $500 every month. After one year, what is the final balance of Bob's student loan?

```
finance_solve(12,5,-50000,500,x,12)
                        46418.6671483
```

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Matrices

Communication Matrix

Determines the paths which the sender could use to communicate to the receiver. The rows represent the senders, while columns represent receivers.

Syntax

com(Matrix, Start, End)

Where,

Matrix represents the communication matrix

Start represents the sender

End represents the receiver

Example

The communication matrix below shows the direct paths by which messages can be sent between two people in a group of six people, U to Z.

$$\text{sender}\quad \begin{array}{c} \\ U \\ V \\ W \\ X \\ Y \\ Z \end{array}\begin{array}{c} \textit{receiver} \\ \begin{array}{cccccc} U & V & W & X & Y & Z \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Source: VCAA 2019 Further Mathematics Written Examination 1 Section B Module 1 Question 7

In how many ways can Y get a message to W ?

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Example – Continued

$$com\left(\begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix},5,3\right)$$

"Path"	"Receivers"
"EC"	1.
"EDBC"	3.
"EFBC"	3.
"EFAC"	3.
"EFDBC"	4.
"EDFBC"	4.
"EDFAC"	4.
"EFABC"	4.
"EFBAC"	4.
"EDBAC"	4.
"EFDBAC"	5.
"EDFBAC"	5.
"EDFABC"	5.
"Total: 13. "	""

Analyse Dominance

Determines the two-step, and total dominance from the one-step dominance matrix.

Syntax

dom_analyse(Matrix)

Where, *Matrix* represents the one-step dominance matrix.

Example

Five friends, Bhavi (B), Kai (K), Oscar (O), Sian (S) and Xavier (X), played a round-robin table tennis tournament. Each friend played each of the others once. Every game had a winner and a loser.

The one-step dominance matrix constructed from the tournament's results is shown below.

$$\text{winner}\quad \begin{array}{c} \\ B \\ K \\ O \\ S \\ X \end{array}\begin{array}{c} \textit{loser} \\ \begin{matrix} B & K & O & S & X \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

A '1' in this matrix shows that the player named in that row defeated the player named in that column.

For example, the '1' in row 3, column 4 shows that Oscar defeated Sian.

Which one of the following tables shows the number of one-step and two-step dominances accumulated by each player in the tournament?

Source: VCAA 2022 Further Mathematics-NHT Written Examination 1 Section B Module 1 Question 6

$$dom\_analyse\left(\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}\right)$$

▶Results:

"Player"	"Won"	"Lost"
"A"	"B,D"	"C,E"
"B"	"E"	"A,C,D"
"C"	"A,B,D"	"E"
"D"	"B"	"A,C,E"
"E"	"A,C,D"	"B"

▶Dominance:

"Player"	"One"	"Two"	"Total"
"A"	2	2	4
"B"	1	3	4
"C"	3	4	7
"D"	1	1	2
"E"	3	6	9

Dominance Solve

Determines the results matrix based off the one-step and two-step dominances of competitors. The winners are the rows, and the losers are the columns.

Syntax

dom_solve(Step1, Step2)

Where, *Step1* and *Step2* are the one-step and two-step dominances respectively.

Example

Five staff members in Elena's office played a round-robin video game tournament, where each employee played each of the other employees once. In each game there was a winner and a loser.
A table of their one-step and two-step dominances was prepared to summarise the results.

Staff member	One-step dominance	Two-step dominance
Ike (*I*)	3	5
Joelene (*J*)	3	4
Katie (*K*)	1	1
Leslie (*L*)	1	2
Mikki (*M*)	2	4

Consider the results matrix shown below.
A '1' in this matrix shows that the player named in that row defeated the player named in that column.
A '0' in this matrix shows that the player named in that row lost to the player named in that column.

Use all of the information provided to complete the results matrix.

Source: VCAA 2021 Further Mathematics Written Examination 2 Section B Module 1 Question 4

$$dom\_solve\left(\begin{bmatrix}3\\3\\1\\1\\2\end{bmatrix},\begin{bmatrix}5\\4\\1\\2\\4\end{bmatrix}\right)$$

▸Results Matrix:

$$\begin{bmatrix}0&1&1&1&0\\0&0&1&1&1\\0&0&0&1&0\\0&0&0&0&1\\1&0&1&0&0\end{bmatrix}$$

▸Results:

"Player"	"Won"	"Lost"
"A"	"B,C,D"	"E"
"B"	"C,D,E"	"A"
"C"	"D"	"A,B,E"
"D"	"E"	"A,B,C"
"E"	"A,C"	"B,D"

Square Root Matrix

Determines the square root of a matrix which was produced by multiplying two **binary** matrices together.

This can be used for finding the one-step dominance from the two-step dominance.

Syntax

sqrt_mat(Matrix)

Where, *Matrix* represents squared matrix.

Example

A badminton competition is held between four players, Amanda (A), Ben (B), Carlos (C) and Darius (D).
In the competition, each player competes in one game with each of the other three players.
The matrix S^2 below shows the two-step dominance that each player has over the other players.

$$S^2 = winner \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \overset{\textit{loser}}{\begin{array}{cccc} A & B & C & D \end{array}} \begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Source: VCAA 2017 Further Mathematics-NHT Written Examination 1 Section B Module 1 Question 7

Determine the one-step dominance matrix.

$$sqrt\_mat\left(\begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\right)$$

▸Results Matrix:

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

▸Results:

"Player"	"Won"	"Lost"
"A"	"B,C,D"	"None"
"B"	"None"	"A,C,D"
"C"	"B"	"A,D"
"D"	"B,C"	"A"

Outcomes

Determines the possible outcomes and final player rankings based off a given matrix.

Syntax

outcomes(Matrix)

Where, *Matrix* represents the current one-step dominance matrix with variables in place of blank spaces.

Note: You may enter the entire matrix, or just the upper triangle as a string. Both cases are shown below.

Example 1

Five competitors, Andy (A), Brie (B), Cleo (C), Della (D) and Eddie (E), participate in a darts tournament.

Each competitor plays each of the other competitors once only, and each match results in a winner and a loser.

The matrix below shows the results of this darts tournament.

There are still two matches that need to be played.

$$\text{winner}\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \overset{\begin{matrix}\textit{loser}\\ A \quad B \quad C \quad D \quad E\end{matrix}}{\begin{bmatrix} 0 & \ldots & 0 & 1 & 0 \\ \ldots & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & \ldots & 1 \\ 0 & 1 & \ldots & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}}$$

A '1' in the matrix shows that the competitor named in that row defeated the competitor named in that column.

For example, the '1' in row 2, column 3 shows that Brie defeated Cleo.

A '…' in the matrix shows that the competitor named in that row has not yet played the competitor named in that column.

The winner of this darts tournament is the competitor with the highest sum of their one-step and two-step dominances.

Which player, by winning their remaining match, will ensure that they are ranked first by the sum of their one-step and two-step dominances?

Source: VCAA 2020 Further Mathematics Written Examination 1 Section B Module 1 Question 9

$$outcomes\left(\begin{bmatrix} 0 & x & 0 & 1 & 0 \\ x & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & x & 1 \\ 0 & 1 & x & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}\right)$$

"Win"	"BD"	"BC"	"AD"	"AC"
"A"	5	5	1	3
"B"	1	1	1	2
"C"	3	2	1	1
"D"	2	3	1	5
"E"	3	3	1	3

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Example 2

Instead of entering the entire matrix, we can enter just the upper triangle in the same fashion inside a string. You can obtain a string by pressing "Ctrl + ×".

Using the same question as in **Example 1**, we see that the upper triangle going from top to bottom and left to right is given by "x010 101 x1 0".

outcomes("x010101x10")

"Win"	"BD"	"BC"	"AD"	"AC"
"A"	5	5	1	3
"B"	1	1	1	2
"C"	3	2	1	1
"D"	2	3	1	5
"E"	3	3	1	3

Done

Leslie Matrix

Determines the Leslie matrix based off the given fertility and survival rates.

Syntax

leslie_mat(fertility_rates, survival_rates)

Where,

fertility_rates is a list containing the fertility rates of all age groups.

survival_rates is a list containing the survival rates of all age groups except the final age group.

Example

Question 30

Data has been collected on the female population of a species of mammal located on a remote island.

The female population has been divided into three age groups, with the initial population (at the time of data collection), the birth rate, and the survival rate of each age group shown in the table below.

	Age group (years)		
	0–2	2–4	4–6
Initial population	2100	6400	4260
Birth rate	0	1.8	1.2
Survival rate	0.7	0.6	0

The Leslie matrix (L) that may be used to model this particular population is

Source: VCAA 2024 General Mathematics Written Examination 1 Question 30

leslie_mat({0,1.8,1.2},{0.7,0.6})

$$\begin{bmatrix} 0. & 1.8 & 1.2 \\ 0.7 & 0. & 0. \\ 0. & 0.6 & 0. \end{bmatrix}$$

Done

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Leslie Table

Determines the state table up to a specified number of generations.

Syntax

leslie(L, R0, Iter)

Where,

L represents the Leslie matrix which models the population.

R0 represents the initial state matrix.

Iter represents the number of rows in the table you wish to generate.

Example

Question 11 (3 marks)

A population of a native animal species lives near the construction site.

To ensure that the species is protected, information about the initial female population was collected at the beginning of 2023. The birth rates and the survival rates of the females in this population were also recorded.

This species has a life span of 4 years and the information collected has been categorised into four age groups: 0–1 year, 1–2 years, 2–3 years, and 3–4 years.

This information is displayed in the initial population matrix, R_0, and the Leslie matrix, L, below.

$$R_0 = \begin{bmatrix} 70 \\ 80 \\ 90 \\ 40 \end{bmatrix} \qquad L = \begin{bmatrix} 0.4 & 0.75 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}$$

ii. complete the following table, showing the initial female population, and the predicted female population after one year, for each of the age groups. 1 mark

Source: VCAA 2024 General Mathematics Written Examination 2 Question 11

$$leslie\left(\begin{bmatrix} 0.4 & 0.75 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}, \begin{bmatrix} 70 \\ 80 \\ 90 \\ 40 \end{bmatrix}, 1\right)$$

▸State Table:

"Age"	"1"	"2"	"3"	"4"	"Total"
"Initial"	70.	80.	90.	40.	280.
"Gen 1"	124.	28.	56.	45.	253.

▸Long Term Decay:

Common ratio: 0.8832326847

11.67673153% decrease

▸Long Term Proportions:

"Age"	"1"	"2"	"3"	"4"
"Prop."	0.4963	0.2248	0.1781	0.1008

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Steady State

Determines the steady state matrix using the transition matrix and the addition matrix.

Syntax

steady_state(T, B)

Where,

T represents the transition matrix.

B represents the addition matrix.

Example

Question 27

The following transition matrix, *T*, models the movement of a species of bird around three different locations, *M*, *N* and *O* from one day to the next.

$$T = \begin{array}{c} \textit{this day} \\ \begin{array}{ccc} M & N & O \end{array} \\ \begin{bmatrix} \frac{1}{3} & 0 & \frac{9}{10} \\ \frac{1}{3} & 1 & \frac{1}{10} \\ \frac{1}{3} & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} M \\ N \\ O \end{array} \quad \textit{next day}$$

Which one of the following statements best represents what will occur in the long term?

A. No birds will remain at location *M*.

B. No birds will remain at location *N*.

C. All of the birds will end up at location *M*.

D. All of the birds will end up at location *O*.

E. An equal number of birds will be at all three locations.

Source: VCAA 2023 General Mathematics Written Examination 1 Question 27

$$steady\_state\left(\begin{bmatrix} \frac{1}{3} & 0 & \frac{9}{10} \\ \frac{1}{3} & 1 & \frac{1}{10} \\ \frac{1}{3} & 0 & 0 \end{bmatrix}, 0\right) \quad \begin{bmatrix} 0. \\ 1. \\ 0. \end{bmatrix}$$

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Transition Table

Determines the state table up to the specific number of generations using the initial state matrix.

Syntax

transition(T, R0, B, Iter)

Where,

T represents the transition matrix.

R0 represents the initial state matrix.

B represents the addition matrix.

Iter represents the number of rows in the table you wish to generate.

Example

To encourage more construction workers (C) to stay, the construction company has given workers an incentive to move into the job of foreman (F).

Matrix R below shows the ways in which staff are expected to change their jobs from year to year with this new incentive in place.

$$R = \begin{array}{c} \textit{this year} \\ \begin{array}{cccc} C & F & M & L \end{array} \\ \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.2 & 0.4 & 0.3 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ C \\ F \\ M \\ L \end{array} \quad \textit{next year}$$

The site always requires at least 330 construction workers.

To ensure that this happens, the company hires an additional 190 construction workers (C) at the beginning of 2024 and each year thereafter.

The matrix V_{n+1} will then be given by

$V_{n+1} = RV_n + Z$, where

$$V_0 = \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix} \begin{matrix} C \\ F \\ M \\ L \end{matrix} \qquad Z = \begin{bmatrix} 190 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} C \\ F \\ M \\ L \end{matrix}$$ and n is the number of years after 2023.

c. How many more staff are there on the site in 2024 than there were in 2023? 1 mark

d. Based on this new model, the company has realised that in the long term there will be more than 200 foremen on site.

In which year will the number of foremen first be above 200? 1 mark

Source: VCAA 2024 General Mathematics Written Examination 2 Question 12

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Example – Continued

$$transition\left(\begin{bmatrix}0.4 & 0.2 & 0 & 0\\ 0.4 & 0.2 & 0.4 & 0\\ 0 & 0.2 & 0.3 & 0\\ 0.2 & 0.4 & 0.3 & 1\end{bmatrix},\begin{bmatrix}330\\50\\10\\0\end{bmatrix},\begin{bmatrix}190\\0\\0\\0\end{bmatrix},4\right)$$

▶State Table:

"Group"	"A"	"B"	"C"	"D"	"Total"
"Initial"	330.	50.	10.	0.	390.
"Iter 1"	332.	146.	13.	89.	491.
"Iter 2"	352.	167.2	33.1	217.7	552.3
"Iter 3"	364.24	187.48	43.37	364.91	595.09
"Iter 4"	373.19	200.54	50.51	525.76	624.24

▶Death/Absent Group Detected

Total does not include Group D

▶System's stability:

System does not stabilise

Inverse Transition

Determines the state table from the given current generation, current state matrix, transition matrix, and addition matrix.

Syntax

transition_inv(T, Rn, B, Iter)

Where,

T represents the transition matrix.

Rn represents the current state matrix.

B represents the addition matrix.

Iter represents the current generation.

Example

Question 32

A large sporting event is held over a period of four consecutive days: Thursday, Friday, Saturday and Sunday.

People can watch the event at four different sites throughout the city: Botanical Gardens (*G*), City Square (*C*), Riverbank (*R*) or Main Beach (*M*).

Let S_n be the state matrix that shows the number of people at each location *n* days after Thursday.

The expected number of people at each location can be determined by the matrix recurrence rule

$$S_{n+1} = TS_n + A$$

where

$$T = \begin{array}{c} \textit{this day} \\ \begin{array}{cccc} G & C & R & M \end{array} \\ \begin{bmatrix} 0.4 & 0.2 & 0.4 & 0 \\ 0.4 & 0.1 & 0.3 & 0.3 \\ 0.1 & 0.4 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 & 0.5 \end{bmatrix} \end{array} \begin{array}{l} G \\ C \\ R \\ M \end{array} \textit{next day} \quad \text{and} \quad A = \begin{bmatrix} 300 \\ 200 \\ 100 \\ 300 \end{bmatrix} \begin{array}{l} G \\ C \\ R \\ M \end{array}$$

Given the state matrix

$$S_3 = \begin{bmatrix} 5620 \\ 6386 \\ 4892 \\ 6902 \end{bmatrix} \begin{array}{l} G \\ C \\ R \\ M \end{array}$$

the number of people watching the event at the Botanical Gardens (*G*) from Thursday to Sunday has

Source: VCAA 2024 General Mathematics Written Examination 1 Question 32

$$transition\_inv\left(\begin{bmatrix}0.4 & 0.2 & 0.4 & 0\\0.4 & 0.1 & 0.3 & 0.3\\0.1 & 0.4 & 0.1 & 0.2\\0.1 & 0.3 & 0.2 & 0.5\end{bmatrix},\begin{bmatrix}5620\\6386\\4892\\6902\end{bmatrix},\begin{bmatrix}300\\200\\100\\300\end{bmatrix},3\right)$$

▸State Table:

"Group"	"A"	"B"	"C"	"D"	"Total"
"Initial"	4924.	4732.	6540.	4904.	21100.
"Iter 1"	5832.	6076.	4120.	5972.	22000.
"Iter 2"	5496.	6168.	4720.	6516.	22900.
"Iter 3"	5620.	6386.	4892.	6902.	23800.

Done

Networks and Decision Mathematics

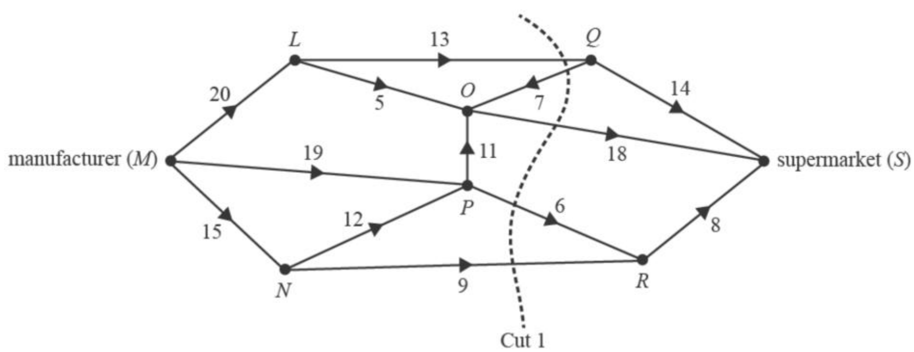
Introduction

This section will discuss how to input networks for the UDFs to use. It is important to know the difference between the two types of graphs (vertex-labelled & edge-labelled) to be able to properly use the UDFs. Furthermore, it is highly recommended that you practice inputting these on your handheld CAS as this is what you will be doing in the end of year VCE exam.

Vertex Labelled Graphs (VLG)

As the name suggests, these are graphs where the vertices are assigned labels (such as A, B, C, etc) rather than the edges. These are common for Dijkstra's algorithm, and flow questions.

An example of a vertex labelled graph is shown below.



Source: VCAA 2024 VCE General Mathematics Examination 2 Question 14

As you can see from the graph above, the vertices of the graph have been labeled with the letters (O, M, N, etc), while the edges are unlabelled.

Syntax

label transitions

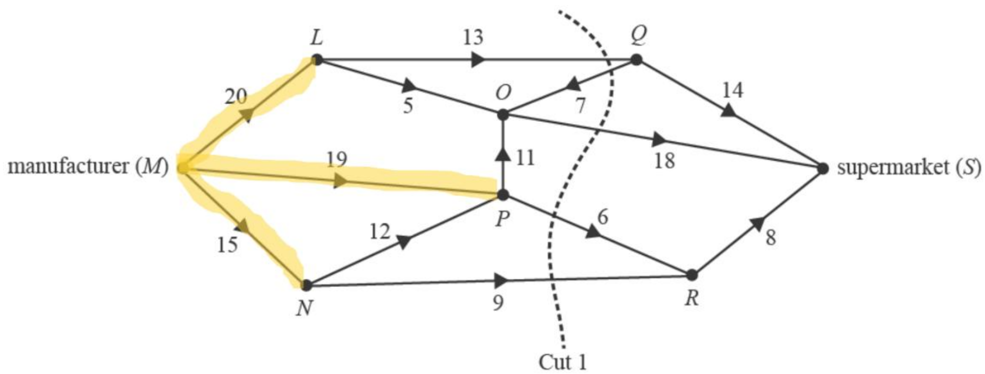
Where,

label represents the letter label of the current vertex

transitions represent the vertices which can be reached from the current vertex, and the associated weight

Example 1

Using the previous graph, suppose we start off at vertex M. We have three possible transitions as shown in the diagram below.

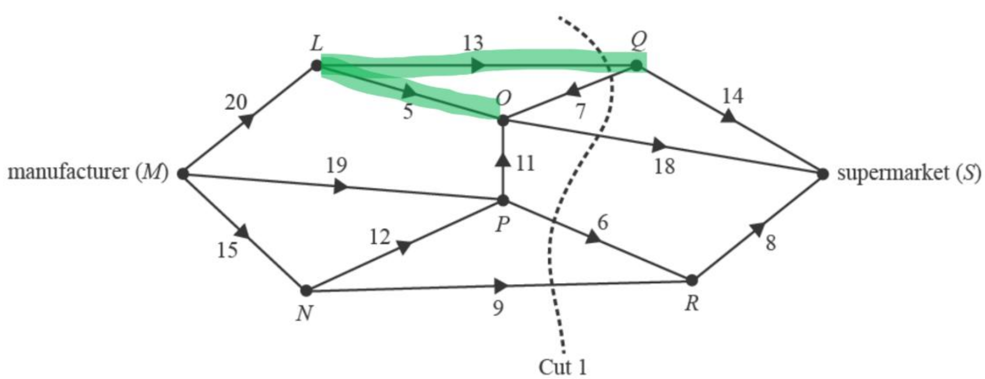


After identifying our transitions, we can use our *label transitions* syntax to write our vertex. Here, we would use

m l20 p19 n15

This is because '*m*' is the label of vertex M, and from vertex M we can transition to either '*l*' with a cost of 20, '*p*' with a cost of 19, and finally '*n*' with a cost of 15.

Now suppose we are at vertex L. We have two possible transitions as shown in the diagram below.



Using the same syntax as before, we would obtain

l q13 o5

This is because '*l*' is the label of the vertex L, and from vertex L we can transition to either '*q*' with a cost of 13, and '*o*' with a cost of 5.

Example 1 – Continued

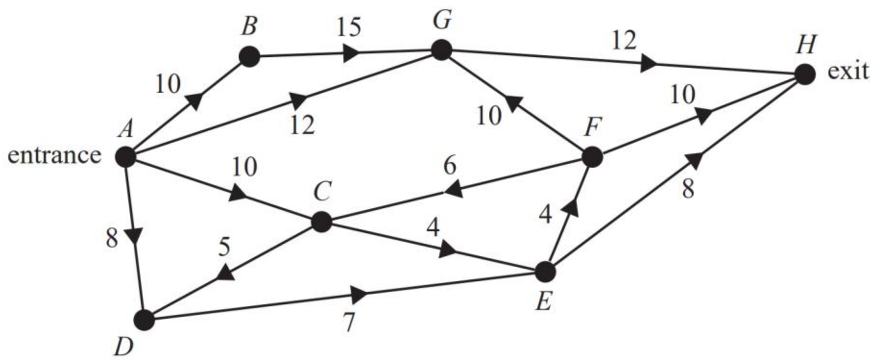
Using the same syntax for the rest of the vertices in the graph, we obtain.

{ml20p19n15, lq13o5, np12r9, os18, po11r6, qo7s14, rs8, s}

Note: Spaces have been omitted to show what should be entered into the CAS.

After completing the list, you can save this into the CAS under a variable for later use. This will then allow you to use this graph for the UDFs.

Example 2



Source: VCAA 2023 General Mathematics Examination 1 Question 39 & Question 40

To practice the syntax, have a go at this one yourself! The solution is shown at the bottom of the page.

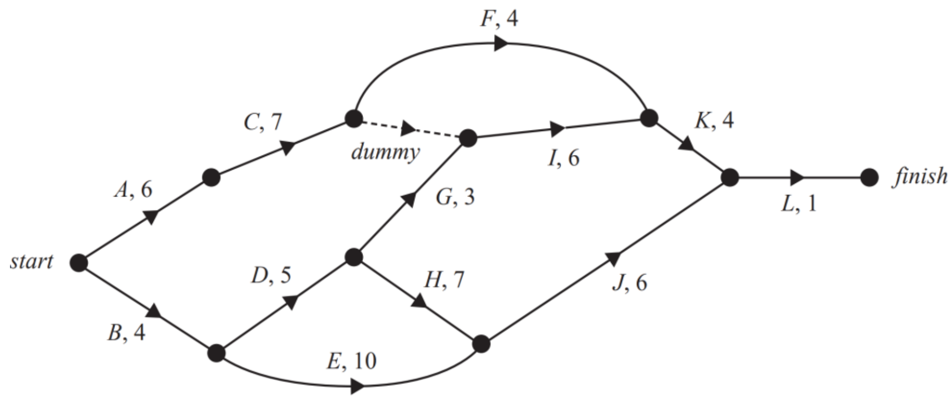
Solution

{ab10g12c10d8, bg15, cd5e4, de7, ef4h8, fg10c6h10, gh12, h}

Edge Labelled Graphs (ELG)

These are graphs where the edges are labelled, while the vertices are unlabelled. These are common for crashing and float time questions.

An example of an edge labelled graph is shown below.



Source: VCAA 2023 General Mathematics Examination 2 Question 14

As you can see from the graph above, the edges are labelled with letters (A, B, C, etc), while the vertices are unlabelled.

Since the edges of these graphs are used to represent activities in general mathematics, they will be referred to as activities in this guide.

Syntax

label duration predecessors

Where,

label represents the letter used to represent the activity

duration represents the value of the activity

predecessors represent the predecessors of the activity

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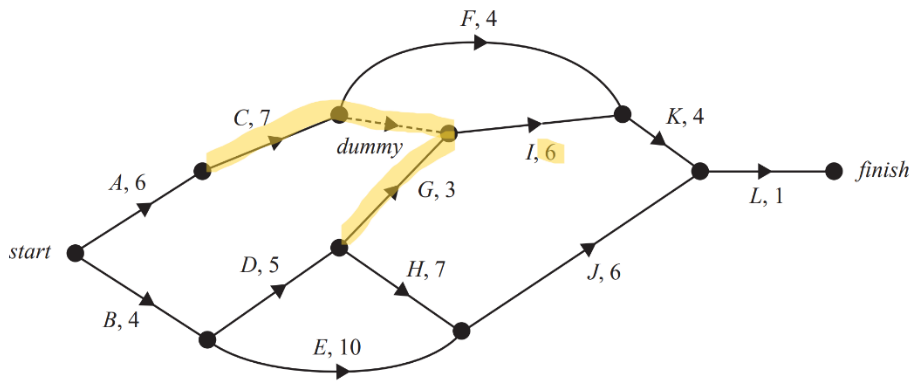
Example 1

Using the graph, suppose we start at activity I, we have two predecessor activities (C and G), and a duration of 6 as shown below.

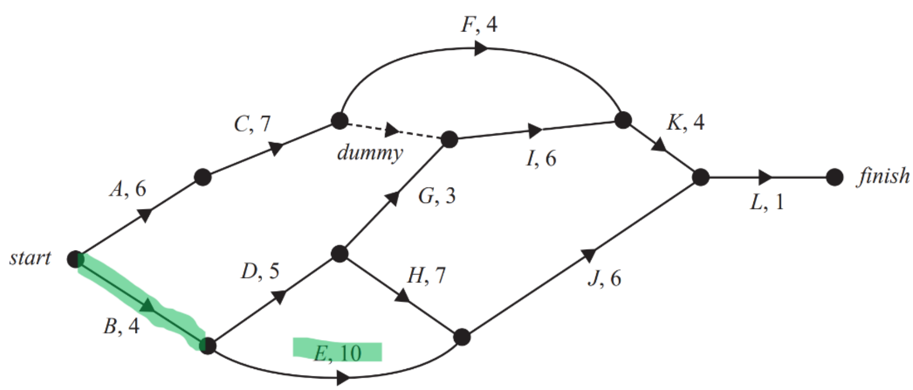


After identifying the duration and predecessors, we can use our *label duration predecessors* syntax to write the activity. Here, we would have

i 6 c g

This is because the label for our activity is '*i*', it has a duration of '6', and its predecessors are '*c*' and '*g*'.

Now suppose we are at activity E. We have one predecessor and a duration of 10 as shown in the diagram below.



Using the same syntax as before, we would obtain

e 10 b

This is because '*e*' is the label of the activity E, activity E has a duration of 10, and the only predecessor of activity E is activity B.

Example 1 – Continued

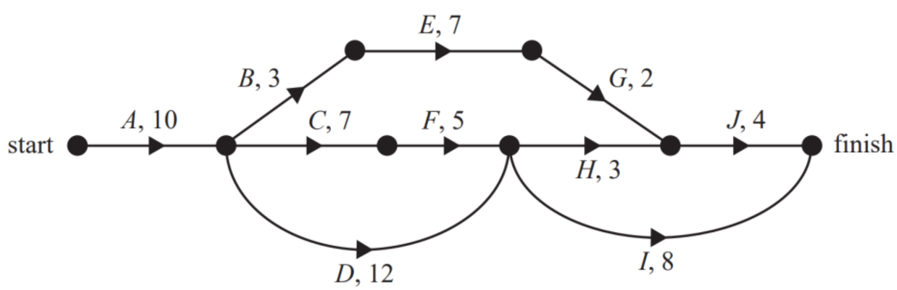
Using the same syntax for the rest of the activities in the graph, we obtain.

{a6, b4, c7a, d5b, e10b, f4c, g3d, h7d, i6gc, j6hk, k4if, l1kj}

Note: Spaces have been omitted to show what should be entered into the CAS.

After completing the list, you can save this into the CAS under a variable for later use. This will then allow you to use this graph for the UDFs.

Example 2



Source: VCAA 2022 General Mathematics Examination 2 Question 2

To practice the syntax, have a go at this one yourself! The solution is shown at the bottom of the page.

Solution

{a10, b3a, c7a, d12a, e7b, f5c, g2e, h3df, i8df, j4hg}

Dijkstra's Algorithm

Uses Dijkstra's algorithm to determine the shortest pathway in a graph.

Syntax

dijkstra(VLG, Start, End)

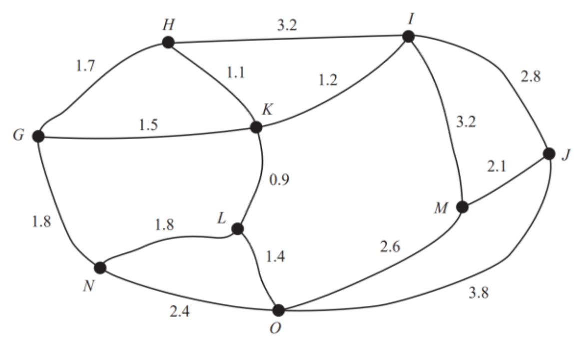
Where,

VLG represents a vertex labelled graph

Start represents the label of the start vertex

End represents the label of the end vertex

Example



Source: VCAA 2023 General Mathematics Examination 2 Question 13

graph {gh17n18k15,hg17k11i32,ih32k12j28m32,ji28m21o38,kh11g15i12l9,ln18k9o14,mj21i32o26,ng18l18o24,on24l14m26j38}

dijkstra(graph,g,m)

▸Dijkstra:

▸Working:

"Iter"	"Visited"	"G"	"H"	"I"	"J"	"K"	"L"	"M"	"N"	"O"
0.	"G"	0.	∞	∞	∞	∞	∞	∞	∞	∞
1.	"K"	0.	17.	∞	∞	15.	∞	∞	18.	∞
2.	"H"	0.	17.	27.	∞	15.	24.	∞	18.	∞
3.	"N"	0.	17.	27.	∞	15.	24.	∞	18.	∞
4.	"L"	0.	17.	27.	∞	15.	24.	∞	18.	42.
5.	"I"	0.	17.	27.	∞	15.	24.	∞	18.	38.
6.	"O"	0.	17.	27.	55.	15.	24.	59.	18.	38.
7.	"J"	0.	17.	27.	55.	15.	24.	59.	18.	38.
8.	"M"	0.	17.	27.	55.	15.	24.	59.	18.	38.

▸Result:

Minimum Distance: 59.

Optimal Path: GKIM

Done

***graph**:={gh17n18k15,hg17k11i32,ih32k12j28m32,ji28m21o38,kh11g15i12l9,ln18k9o14,mj21i32o26,ng18l18o24,on24l14m26j38}*

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Prim’s Algorithm

Uses Prim’s Algorithm to find a minimum spanning tree of the graph.

Syntax

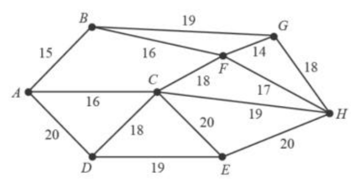
prim(VLG)

Where,

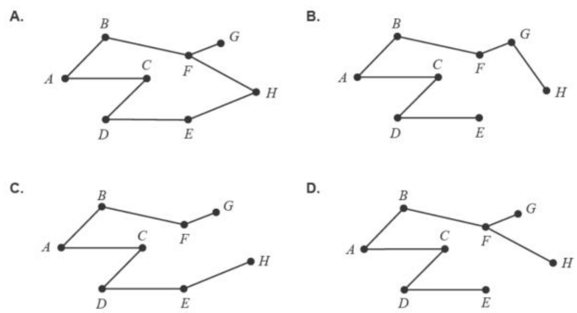
VLG represents a vertex labelled graph

Question 36

Eight houses in an estate are to be connected to the internet via underground cables.
The network below shows the possible connections between the houses.
The vertices represent the houses.
The numbers on the edges represent the length of cable connecting pairs of houses, in metres.



The graph that represents the minimum length of cable needed to connect all the houses is



Source: VCAA 2024 General Mathematics Examination 1 Question 36

graph	{*ab15c16d20,bg19f16,ca16f18h19e20d18,da20c18e19,ed19c20h20,fb16g14c18h17,gb19f14h18,hg18f17c19e20*}
prim(graph)	

▸Working:

"Edge"	"Length"
"AB"	15.
"BF"	16.
"FG"	14.
"AC"	16.
"FH"	17.
"CD"	18.
"DE"	19.

▸Solution:
115.
Warning: This program has not been extensively tested.

Done

***graph*:=**{*ab15c16d20,bg19f16,ca16f18h19e20d18,da20c18e19,ed19c20h20,fb16g14c18h17,gb19f14h18,*

hg18f17c19e20}

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Hamiltonian

Finds the minimum Hamiltonian cycle or path in a graph. If a Hamiltonian cycle or path cannot be found it returns ‘false’.

Syntax

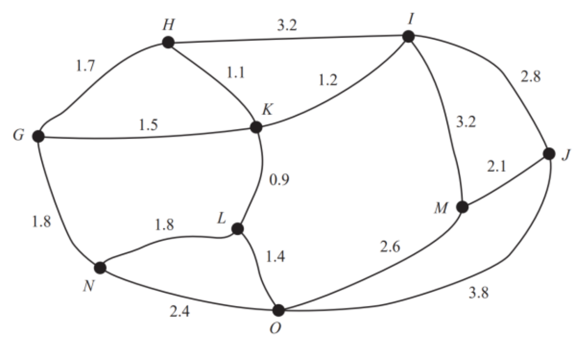
hamiltonian(VLG, Start, End)

Where,

VLG represents a vertex labelled graph

Start represents the label of the start vertex

End represents the label of the end vertex



b. Reynold would like to visit all the landmarks and return to *G*.

Write down a route that Reynold could follow to minimise the total distance travelled. 1 mark

Source: VCAA 2023 General Mathematics Examination 2 Question 13

```
graph    {gh17n18k15,hg17k11i32,ih32k12j28m32,ji28m21o38,kh11g15i12l9,ln18k9o14,mj21i32o26,ng18l18o24,on24l14m26j38}
hamiltonian(graph,g,g)
                                   ["Type"     "Nodes"        "Cost"]
                                   ["Cycle"    "GHKIJMOLNG"   169   ]
                                   Warning: This program has not been extensively tested.
                                                                              Done
```

graph*:={gh17n18k15,hg17k11i32,ih32k12j28m32,ji28m21o38,kh11g15i12l9,ln18k9o14,mj21i32o26,ng18l18o24,on24l14m26j38}*

Eulerian

Finds the minimum Eulerian trail or circuit, if possible. Otherwise, it determines the minimum trail which traverses all edges.

Syntax

eulerian(VLG, Start, End)

Where,

VLG represents a vertex labelled graph

Start represents the label of the start vertex

End represents the start vertex (for circuit), or a blank string (for trail)

Example

c. Shyla would like to travel along all the roads.
To complete this journey in the minimum distance, she will travel along two roads twice.
Shyla will leave from landmark *G* but end at a different landmark.

Complete the following by filling in the boxes provided. 1 mark

The two roads that will be travelled along twice are the roads between:

- vertex ☐ and vertex ☐
- vertex ☐ and vertex ☐

Source: VCAA 2023 General Mathematics Examination 2 Question 13

graph {gh17n18k15,hg17k11i32,ih32k12j28m32,ji28m21o38,kh11g15i12l9,ln18k9o14,mj21i32o26,ng18l18o24,on24l14m26j38}

eulerian(graph,g,"")

▸Repeated Edges:

"Edge"	"Cost"
"JM"	21.
"LN"	18.

▸Total Cost:
354.
▸Trail:
GHIJMIKGNLNOJMOLKH
Warning: This program has not been extensively tested.

Done

graph*:={gh17n18k15,hg17k11i32,ih32k12j28m32,ji28m21o38,kh11g15i12l9,ln18k9o14,mj21i32o26,ng18l18o24,on24l14m26j38}*

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Float Time

Determines the float times for all activities in an edge labelled graph.

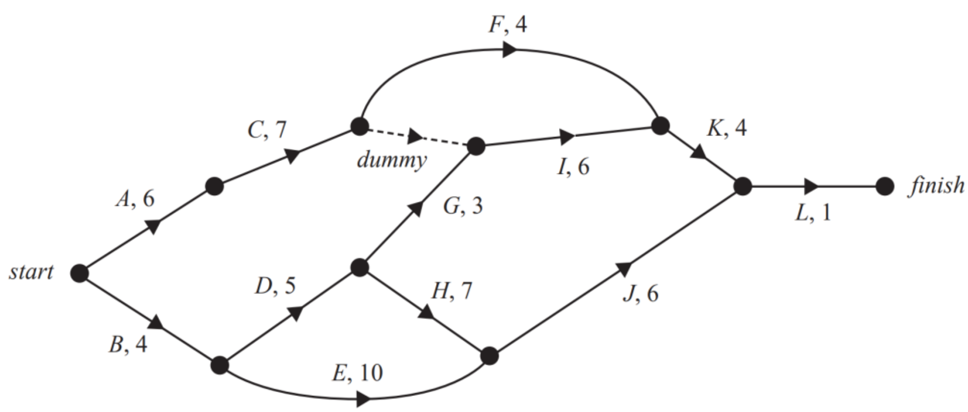
Syntax

float_time(ELG)

Where,

ELG is an edge labelled graph

Example



Source: VCAA 2023 General Mathematics Examination 2 Question 13

c. How many activities have a float time of zero? 1 mark

graph:={a6,b4,c7a,d5b,e10b,f4c,g3d,h7d,i6cg,j6he,k4fi,l1jk} {a6,b4,c7a,d5b,e10b,f4c,g3d,h7d,i6cg,j6he,k4fi,l1jk}

float_time(graph)

▸Float Time:

"Activity"	"A"	"B"	"C"	"D"	"E"	"F"	"G"	"H"	"I"	"J"	"K"	"L"	"Finish"
"EST"	0.	0.	6.	4.	4.	13.	9.	9.	13.	16.	19.	23.	24.
"LST"	0.	1.	6.	5.	7.	15.	10.	10.	13.	17.	19.	23.	24.
"Float"	0.	1.	0.	1.	3.	2.	1.	1.	0.	1.	0.	0.	0.

▸Non−Critical Acitivities:

{"B","D","E","F","G","H","J"}

Total: 7.

▸Critical Activities:

{"A","C","I","K","L"}

Total: 5.

▸Critical Path(s):

ACIKL

▸Critical Time:

24.

Done

graph*:={a6,b4,c7a,d5b,e10b,f4c,g3d,h7d,i6cg,j6he,k4fi,l1jk}*

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Dummy Activity

Determines the number and location of dummy activities for a given project network.

Syntax

dummy(ELG)

Where, *ELG* represents an edge labelled graph

Example

The project requires 11 activities, *A* to *K*.

Determine the number of dummy activities required and where they occur.

Activity	Immediate predecessor(s)
A	–
B	–
C	–
D	*A*
E	*B*, *F*
F	*C*
G	*C*
H	*D*, *E*
I	*F*
J	*E*, *G*, *I*
K	*H*, *J*

Source: VCAA 2022 Further Mathematics-NHT Written Examination 1 Section B Module 2 Question 8

graph {*a*,*b*,*c*,*da*,*ebf*,*fc*,*gc*,*hde*,"if",*jegi*,*khj*}

dummy(*graph*)

▸Dummy Activities:

"End"	"Start"
"E"	"H"
"E"	"J"
"F"	"E"
"Total"	3.

Warning: This program assumes multiple distinct activities can start and end at the same nodes.

Done

graph*:=*{a,b,c,da,ebf,fc,gc,hde,"if",jegi,khj}

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Project Crashing

Determines the reductions in the project's activities which result in the minimum cost within a specified budget.

Syntax

crashing(ELG, Tasks, Reduction(s), Budget)

Where,

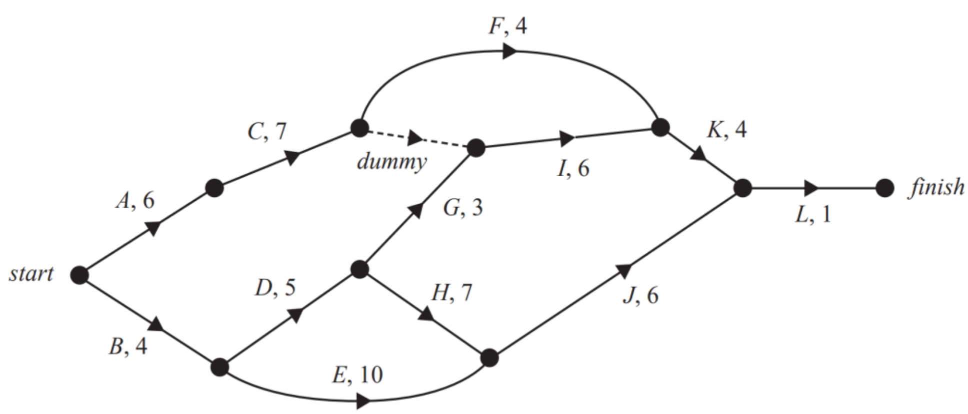
ELG represents an edge labelled graph

Tasks represents either a matrix or a list containing the activity labels

Reduction(s) represents either a number or a list used to specify the maximum reduction of each activity

Budget represents the available budget

Example 1



Example 1 – Continued

The managers of the project are able to reduce the time, in days, of six activities.
These reductions will result in an increase in the cost of completing the activity.
The maximum decrease in time of any activity is two days.

Activity	*A*	*B*	*F*	*H*	*I*	*K*
Daily cost ($)	1500	2000	2500	1000	1500	3000

e. The managers of the project have a maximum budget of $15 000 to reduce the time for several activities to produce the maximum reduction in the project's overall completion time.

Complete the table below, showing the reductions in individual activity completion times that would achieve the earliest completion time within the $15 000 budget. 1 mark

Activity	**Reduction in completion time (0, 1 or 2 days)**
A	
B	
F	
H	
I	
K	

Source: VCAA 2023 General Mathematics Examination 2 Question 13

graph $\{a6,b4,c7a,d5b,e10b,f4c,g3d,h7d,i6cg,j6he,k4fi,l1jk\}$

$$crashing\left(graph, \begin{bmatrix} a & 1500. \\ b & 2000. \\ f & 2500. \\ h & 1000. \\ i & 1500. \\ k & 3000. \end{bmatrix}, 2, 15000\right)$$

▸Working:

"Path"	"Start"	"↓A"	"↓I"	"↓H"	"↓I"	"↓H"	"↓B"	"↓A"	"↓B"	"↓K"
"BDGIKL"	23.	23.	22.	22.	21.	21.	20.	20.	19.	18.
"ACIKL"	24.	23.	22.	22.	21.	21.	21.	20.	20.	19.
"ACFKL"	22.	21.	21.	21.	21.	21.	21.	20.	20.	19.
"BEJL"	21.	21.	21.	21.	21.	21.	20.	20.	19.	19.
"BDHJL"	23.	23.	23.	22.	22.	21.	20.	20.	19.	19.
"□"	"□"	"□"	"□"	"□"	"□"	"□"	"□"	"□"	"□"	"□"
"Crit Time"	24.	23.	23.	22.	22.	21.	21.	20.	20.	19.
"Min Cost"	0.	1500.	3000.	4000.	5500.	6500.	8500.	10000.	12000.	15000.

▸Solution:

"Cost"	"Time"	"A"	"B"	"F"	"H"	"I"	"K"
15000.	19.	2.	2.	0.	2.	2.	1.

Done

graph:=$\{a6,b4,c7a,d5b,e10b,f4c,g3d,h7d,i6cg,j6he,k4fi,l1jk\}$

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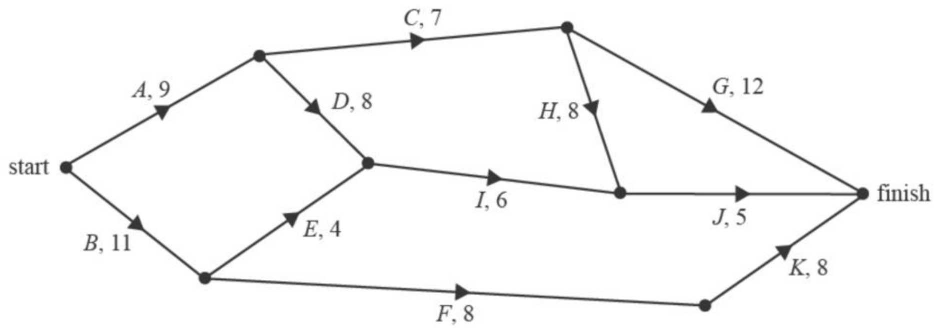
Example 2



e. The owners of the supermarket want the project completed earlier.

They will pay to reduce the time of some of the activities.

A reduction in completion time of an activity will incur an additional cost of \$10 000 per week.

Activities can be reduced by a maximum of two weeks.

The minimum number of weeks an activity can be reduced to is seven weeks.

What is the minimum amount the owners of the supermarket will have to pay to reduce the completion time of the project as much as possible? 1 mark

____

Source: <u>VCAA 2024 General Mathematics Examination 2 Question 13</u>

graph:={a9,b11,c7a,d8a,e4b,f8b,g12cde,h8cde,i6de,j5hi,k8f} {a9,b11,c7a,d8a,e4b,f8b,g12cde,h8cde,i6de,j5hi,k8f}

crashing(*graph*,{a,b,d,f,h,g,k,10000},{2,2,1,1,1,2,1},∞)

▸Working:

"Path"	"Start"	"↓A"	"↓A"	"↓H"	"↓B"	"↓D"
"BFK"	27.	27.	27.	27.	26.	26.
"BEIJ"	26.	26.	26.	26.	25.	25.
"ADIJ"	28.	27.	26.	26.	26.	25.
"BEHJ"	28.	28.	28.	27.	26.	26.
"ADHJ"	30.	29.	28.	27.	27.	26.
"ACHJ"	29.	28.	27.	26.	26.	26.
"BEG"	27.	27.	27.	27.	26.	26.
"ADG"	29.	28.	27.	27.	27.	26.
"ACG"	28.	27.	26.	26.	26.	26.
"⬚"	"⬚"	"⬚"	"⬚"	"⬚"	"⬚"	"⬚"
"Crit Time"	30.	29.	28.	27.	27.	26.
"Min Cost"	0.	10000.	20000.	30000.	40000.	50000.

▸Solution:

"Cost"	"Time"	"A"	"B"	"D"	"F"	"H"	"G"	"K"
50000.	26.	2.	1.	1.	0.	1.	0.	0.

Done

***graph*:=**{a9,b11,c7a,d8a,e4b,f8b,g12cde,h8cde,i6de,j5hi,k8f}

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Flow

Determines the maximum flow through a vertex labelled network, the edges within the minimum cut, the edges which can be revered to increase flow, as well as the minimum values of these edges required to provide a maximal increase in flow.

Syntax

flow(ELG, Source, Sink)

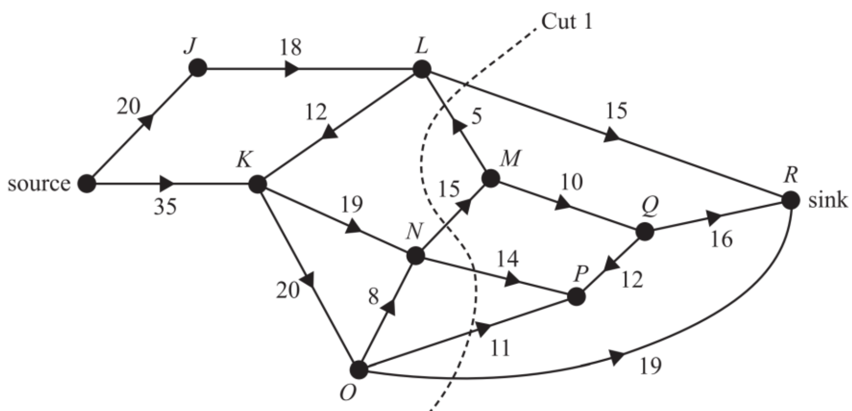
Where,

ELG is an edge labelled graph

Source is the source of the network

Sink is the sink of the network

Example



b. What is the maximum flow of stormwater, in litres per minute, from the source to the sink? 1 mark

c. The direction of flow is reversed through one pipe.

Complete the following sentence by filling in the boxes provided. 1 mark

The pipe that should have its flow reversed to cause the largest increase in flow from source to sink is the pipe that runs from vertex ☐ to vertex ☐.

Example – Continued

d. The capacity of one pipe is increased.

Complete the following sentences by filling in the boxes provided. 1 mark

The pipe that should have its capacity increased to cause the largest increase in flow from source to sink is the pipe that runs from vertex ☐ to vertex ☐. Its new capacity, in litres per minute, should be at least ☐.

Source: VCAA 2022 Further Mathematics Written Examination 2 Question 3

graph {*sj20k35,jl18,kn19o20,lk12r15,mq10l5,nm15p14,on8p11r19,p,qr16p12,r0*}

flow(*graph,s,r*)

▸Maximum Flow:

44.

▸Minimum Cut:

"Edge"	"Cap"	"New Flow"	"ΔFlow"
"LR"	23.	52.	8.
"MQ"	15.	49.	5.
"OR"	20.	45.	1.

▸Reversible Edges:

"Edge"	"New Flow"	"ΔFlow"
"QP"	50.	6.

Done

graph*:=*{sj20k35,jl18,kn19o20,lk12r15,mq10l5,nm15p14,on8p11r19,p,qr16p12,r0}

Note

Cap refers to the minimum weight the edge can have to cause a maximal increase in flow from source to sink

New Flow refers to the maximum new flow which can be produced by increasing the weight of the edge

Δ*Flow* refers to the increase in flow

Path Flow

Determines the flow along each path of the graph.

Syntax

path_flow(VLG, Source, Sink)

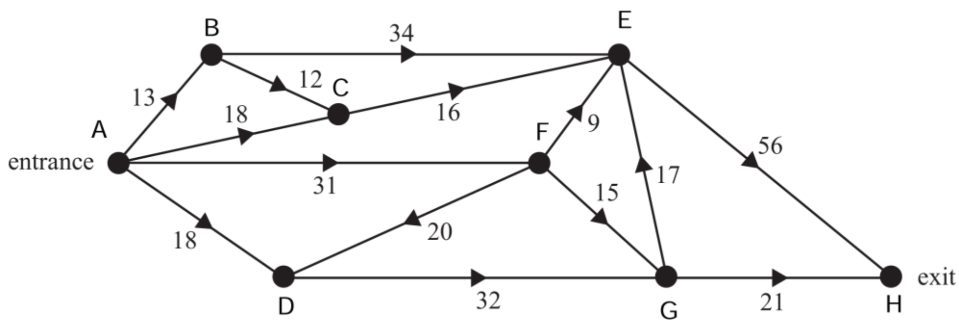
Where,

VLG represents a vertex labelled graph

Source represents the source of the network

Sink represents the sink of the network

Example



Source: VCAA 2024 General Mathematics-NHT Written Examination 1 Question 38 and Question 39

Determine the path in the graph with the greatest flow.

graph {*ab13c18d18f31,bc12e34,ce16,dg32,eh56,fe9d20g15,ge17h21,h*}

path_flow(graph,a,h)

"Path"	"Flow"
"AFDGH"	20
"ADGH"	18
"ADGEH"	17
"AFDGEH"	17
"ACEH"	16
"AFGEH"	15
"AFGH"	15
"ABEH"	13
"ABCEH"	12
"AFEH"	9
"Total: 10"	""

Done

graph:=*{ab13c18d18f31,bc12e34,ce16,dg32,eh56,fe9d20g15,ge17h21,h}*

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Allocate

Determines all optimal tasks allocations which result in a minimum completion time.

Syntax

allocate(Cost Matrix)

Where, *Cost Matrix* represents the matrix containing workers as its rows, and the time each take to complete a task in its columns

Example

	Alan	Brianna	Chamath	Deidre	Ewen
Job 1	5	8	5	8	7
Job 2	5	7	6	7	4
Job 3	9	5	7	5	9
Job 4	7	7	9	8	5
Job 5	4	4	4	4	3

Source: VCAA 2016 Further Mathematics Examination 1 Section B Module 2 Question 8

Determine all allocations which allow the five jobs to be completed in the minimum amount of time.

cost_matrix

$$\begin{bmatrix} 5. & 5. & 9. & 7. & 4. \\ 8. & 7. & 5. & 7. & 4. \\ 5. & 6. & 7. & 9. & 4. \\ 8. & 7. & 5. & 8. & 4. \\ 7. & 4. & 9. & 5. & 3. \end{bmatrix}$$

allocate(cost_matrix)

▸Allocation 1. :

"Worker"	"Task"	"Duration"
1.	2.	5.
2.	3.	5.
3.	1.	5.
4.	5.	4.
5.	4.	5.

▸Allocation 2. :

"Worker"	"Task"	"Duration"
1.	2.	5.
2.	5.	4.
3.	1.	5.
4.	3.	5.
5.	4.	5.

▸Minimum cost: 24.

***cost_matrix**:=[[5,5,9,7,4][8,7,5,7,4][5,6,7,9,4][8,7,5,8,4][7,4,9,5,3]]*

Hungarian Algorithm

Uses the Hungarian algorithm to determine the optimal allocations to produce the minimum cost. The rows of the matrix represent the workers, while the columns represent the tasks.

Syntax

hungarian(Cost Matrix)

Where, *Cost Matrix* represents the matrix containing workers as its rows, and the time each take to complete a task in its columns

Example

Anush, Blake, Carly, and Dexter are workers on a construction site. They are each allocated one task. The time, in hours, it takes for each worker to complete each task is shown in the table below.

	Task 1	Task 2	Task 3	Task 4
Anush	12	8	16	9
Blake	10	7	15	10
Carly	11	10	18	12
Dexter	10	14	16	11

Source: VCAA 2024 General Mathematics Written Examination 1 Question 39

The tasks must be completed sequentially and in numerical order.

Determine the minimum time, in hours, required to complete all four tasks.

$$cost\_matrix \quad \begin{bmatrix} 12. & 8. & 16. & 9. \\ 10. & 7. & 15. & 10. \\ 11. & 10. & 18. & 12. \\ 10. & 14. & 16. & 11. \end{bmatrix}$$

Example – Continued

hungarian(cost_matrix)

▶Given cost matrix:

"Task"	1.	2.	3.	4.
"Worker 1."	12.	8.	16.	9.
"Worker 2."	10.	7.	15.	10.
"Worker 3."	11.	10.	18.	12.
"Worker 4."	10.	14.	16.	11.

▶Step 1: Row reduction

"Task"	1.	2.	3.	4.	"Min"
"Worker 1."	4.	0.	8.	1.	8.
"Worker 2."	3.	0.	8.	3.	7.
"Worker 3."	1.	0.	8.	2.	10.
"Worker 4."	0.	4.	6.	1.	10.

▶Step 2: Column reduction

"Task"	1.	2.	3.	4.
"Worker 1."	4.	0.	2.	0.
"Worker 2."	3.	0.	2.	2.
"Worker 3."	1.	0.	2.	1.
"Worker 4."	0.	4.	0.	0.
"Min"	0.	0.	6.	1.

▶Step 3. : Update by the min value of 1.

"Task"	1.	2.	3.	4.
"Worker 1."	3.	0.	1.	0.
"Worker 2."	2.	0.	1.	2.
"Worker 3."	0.	0.	1.	1.
"Worker 4."	0.	5.	0.	1.

▶Step 4. : Number of lines equals number of workers, ready for allocation

▶Optimal allocations

"Task"	1.	2.	3.	4.
"Worker 1."	0.	0.	0.	1.
"Worker 2."	0.	1.	0.	0.
"Worker 3."	1.	0.	0.	0.
"Worker 4."	0.	0.	1.	0.

▶Step 5. : Calculate minimum cost

9.+7.+11.+16. =43.

▶Summary:

"Worker"	"Task"	"Duration"
1.	4.	9.
2.	2.	7.
3.	1.	11.
4.	3.	16.

Min Cost = 43.

***cost_matrix**:=[[12,8,16,9][10,7,15,10][11,10,18,12][10,14,16,11]]*

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